EXHIBIT "A"

TRANSPORT PHENOMENA

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by using arguments similar to those in §6.2. Hence from the dimensional analysis of the partial differential equations describing the flow and from the definition of the friction factor we have obtained the result that f may be correlated as a function of Re alone.

Many experimental data have been taken for flow around spheres, so that a chart of f versus Re is available for smooth spheres. (See Fig. 6.3-1.) For this system there is no sharp transition from an unstable laminar flow curve to a stable turbulent flow curve as was indicated for tubes in Fig. 6.2-2 at $Re \doteq 2.1 \times 10^3$. In this system, as the flow rate increases, there is an increase in the amount of eddying behind the sphere. The kink in the curve at about $Re = 2 \times 10^5$ is associated with the shift of the boundary-layer separation zone from in front of the equator to in back of the equator of the sphere.[a]

We have purposely chosen to discuss the sphere immediately after the tube in order to emphasize the fact that various flow systems may behave quite differently. Several points of difference between the two systems are:

For tubes there is a rather well-defined laminar-turbulent transition at $Re \doteq 2 \times 10^3$.	For spheres the f-curve exhibits no well-defined laminar-turbulent transition.
For smooth tubes the only contribution to f is friction drag.	For spheres there are contributions to f owing to both friction drag and form drag.
For tubes there is no boundary layer separation.	For spheres there is a kink in f-curve associated with a shift in the separation zone.

The general shape of the curves in Figs. 6.2-2 and 6.3-1 should be carefully remembered.

For the *creeping flow region*, we already know that the drag force is given by *Stokes's law*, which is a consequence of an analytical solution of the equations of motion and continuity (with the term $\rho Dv/Dt$ omitted from the equation of motion given in Eq. 3.2-20). By rearranging Stokes's law (Eq. 2.6-14) in the form of Eq. 6.1-5, we get

$$F_k = \pi R^2 \cdot \frac{1}{2}\rho v_\infty^2 \cdot \frac{24}{\left(\frac{D v_\infty \rho}{\mu}\right)} \qquad (6.3\text{-}19)$$

Hence, for *creeping flow* around a sphere,

$$f = \frac{24}{Re} \qquad Re < 0.1 \qquad (6.3\text{-}20)$$

This is the straight-line portion of the log f versus log Re curve.

[a] See H. Schlichting, *Boundary Layer Theory*, McGraw-Hill, New York (1955), pp. 34-3:

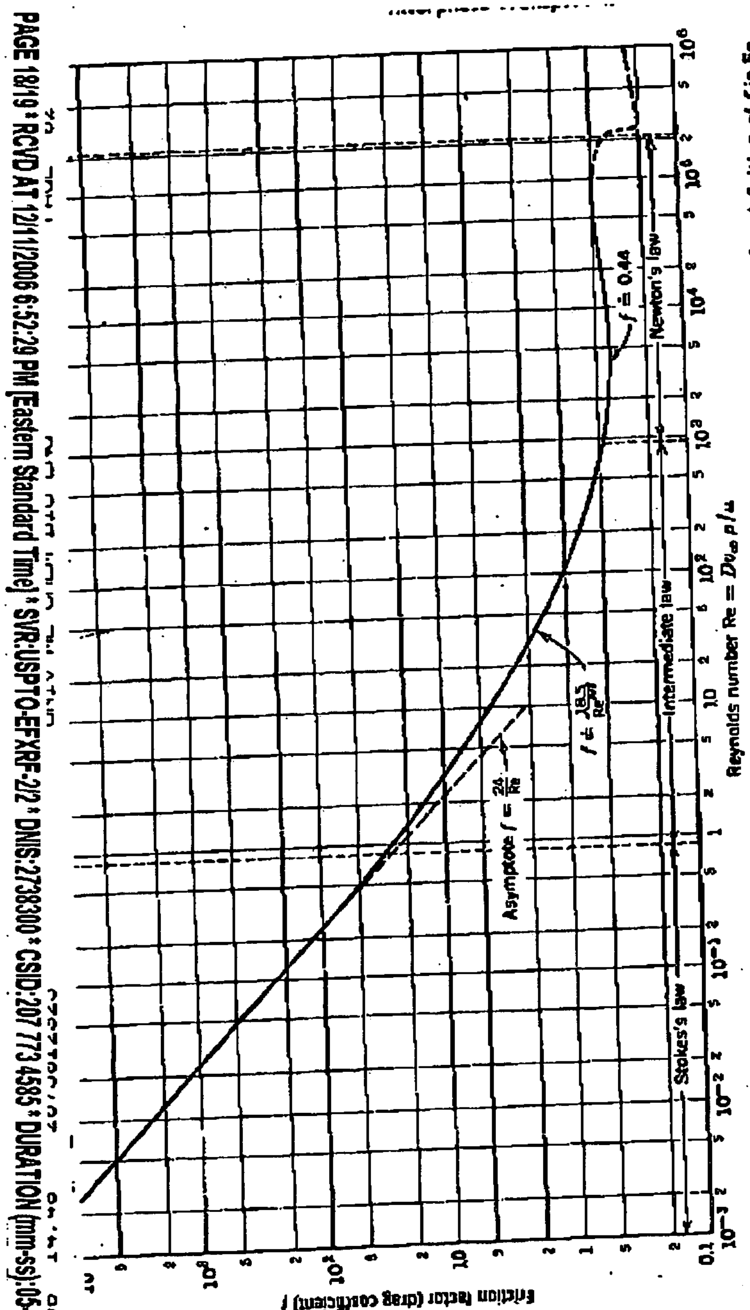

Fig. 6.3-1. Friction factor (or drag coefficient) for spheres moving relative to a fluid with a velocity v_∞. See definition of f in Eq. 6.1-5. [Curve taken from C. E. Lapple, "Dust and Mist Collection," in *Chemical Engineers' Handbook* (ed. by J. H. Perry), McGraw-Hill, New York (1950), Third Edition, p. 1018.]

higher values of the Reynolds number, it is very difficult to make theoretical calculations. Several investigators have managed to te f as far as Re = 10 but only with a considerable amount of effort. the f-curve for Re > 0.1 is a result of experiment. Occasionally, analytical expressions for the higher Reynolds number regions are For the *intermediate region*, we may write very approximately

$$f \doteq \frac{18.5}{\mathrm{Re}^{3/5}} \qquad 2 < \mathrm{Re} < 5 \times 10^{2} \tag{6.3-21}$$

indicates a lesser dependence on Re than in Stokes's law. This ion is less accurate than Stokes's law for Re < 2.

higher Re, we see that the friction factor is approximately constant. known as the *Newton's law region*, for which

$$f \doteq 0.44 \qquad 5 \times 10^{2} < \mathrm{Re} < 2 \times 10^{5} \tag{6.3-22}$$

region the drag force acting on the sphere is approximately propor- to the square of the velocity of the fluid moving past the sphere. on 6.3-22 is a useful approximation for making rapid estimates. hat Newton's "law" for the drag force on a sphere is not to be confused ewton's law of viscosity or Newton's laws of motion.)

y extensions of Fig. 6.3-1 have been made, but a systematic study be beyond the scope of the text. Among the effects investigated are fects (see Problem 6.O), fall of droplets with internal circulation,[3] particles in non-Newtonian fluids,[4] hindered settling (i.e., fall of of particles which interfere with one another),[5] unsteady flow,[6] nspherical particles.[7,8]

ample 6.3-1. Determination of Diameter of a Falling Sphere

spheres of density $\rho_{sph} = 2.62$ g cm^{-3} are allowed to fall through carbon ride ($\rho = 1.59$ g cm^{-3} and $\mu = 9.58$ millipoises) at 20° C in an experiment ring reaction times in making time observations with stopwatches and more e devices. What diameter should the spheres be in order to have a terminal of about 65 cm sec^{-1}?

lmb, *Hydrodynamics*, Dover (1945), Sixth Edition pp. 600–601; S. Hu and R. C. *A.I.Ch.E. Journal*, 1, 42–48 (1955).
Slattery, doctoral thesis, University of Wisconsin (1959).
. Steinour, *Ind. Eng. Chem.*, 36, 618–624, 840–847, 900–901 (1947); see also C. E. *Fluid and Particle Dynamics*, University of Delaware Press, Newark (1951), 13.
Hughes and E. R. Gilliland, *Chem. Eng. Prog.*, 48, 497–504 (1952).
Pettyjohn and E. B. Christiansen, *Chem. Eng. Prog.*, 44, 157 (1948).
Becker, *Can. J. Chem. Eng.* 37, 85–91 (1959).

Solution. To find the sphere diameter, we have to solve Eq. 6.1-7 for D. However, in this equation one has to know D in order to get f; and f is given by the solid curve in Fig. 6.3-1. A trial-and-error procedure can be used, taking $f = 0.44$ as a first guess.

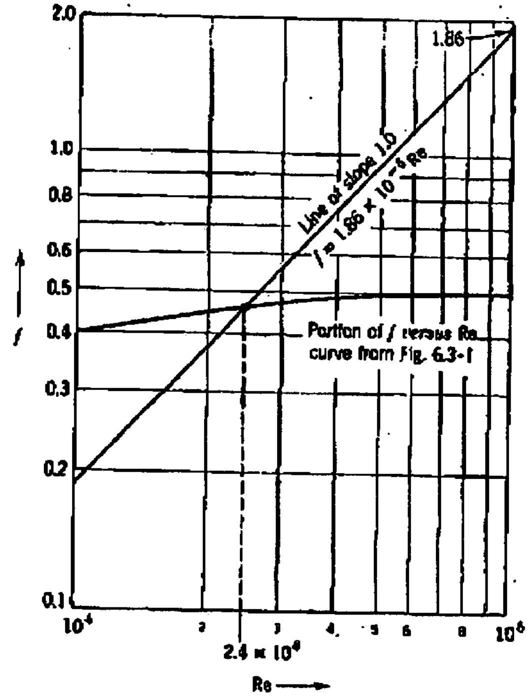

Fig. 6.3-2. Graphical procedure used in Example 6.3-1.

Alternatively, we can solve Eq. 6.1-7 for f and then note that f/Re is a quantity independent of D:

$$\frac{f}{\mathrm{Re}} = \frac{4}{3}\frac{g\mu}{\rho v_\infty^3}\left(\frac{\rho_{sph} - \rho}{\rho}\right) \tag{6.3-23}$$

The quantity on the right side can be calculated with the foregoing data, and we can call it C. Hence we have two simultaneous equations to solve:

$$f = C\mathrm{Re} \qquad \text{(from Eq. 6.3-23)} \tag{6.3-24}$$

$$f = f(\mathrm{Re}) \qquad \text{(given in Fig. 6.3-1)} \tag{6.3-25}$$

Equation 6.3-24 is a straight line of slope unity on the log-log plot of f versus Re. For the problem at hand,

$$C = \frac{4}{3}\frac{(980)(9.58 \times 10^{-3})}{(1.59)(65)^3}\left(\frac{2.62 - 1.59}{1.59}\right) = 1.86 \times 10^{-5} \tag{6.3-26}$$